Understanding power dissipation of piezo-electric stick-slip positioners in cryogenic applications is critical because it can potentially affect experimental conditions. Since the heat load can be expressed in various ways, it is easy to misinterpret data and draw incorrect conclusions. This application note separates and quantifies the different sources of power dissipation; piezoelectric losses, frictional losses, and cable losses. It then compares these contributions for three piezo actuator technologies: a 150 V linear stack, a 60 V linear stack, and a 500 V shear piezo. Benchmark dissipation values are established for a stick-slip positioner operated with a sawtooth drive signal.

## 1. Stick-slip positioner

Stick-slip positioners operate by driving the piezo with a sawtooth-like voltage at pk-pk amplitude $U_{pp}$, equivalent to piezo stroke $S_p$. During the slow ramp $t_0$ - $t_1$ the table sticks to the plate, but during the fast ramp $t_1$ - $t_2$ table inertia will cause it to slip with respect to the fast-retracting plate. After a cycle $t_0$ - $t_1$ - $t_2$ the piezo and plate have returned to their initial positions, but the table has moved a distance $S_p$. Repeating such steps will result in macroscopic motion.

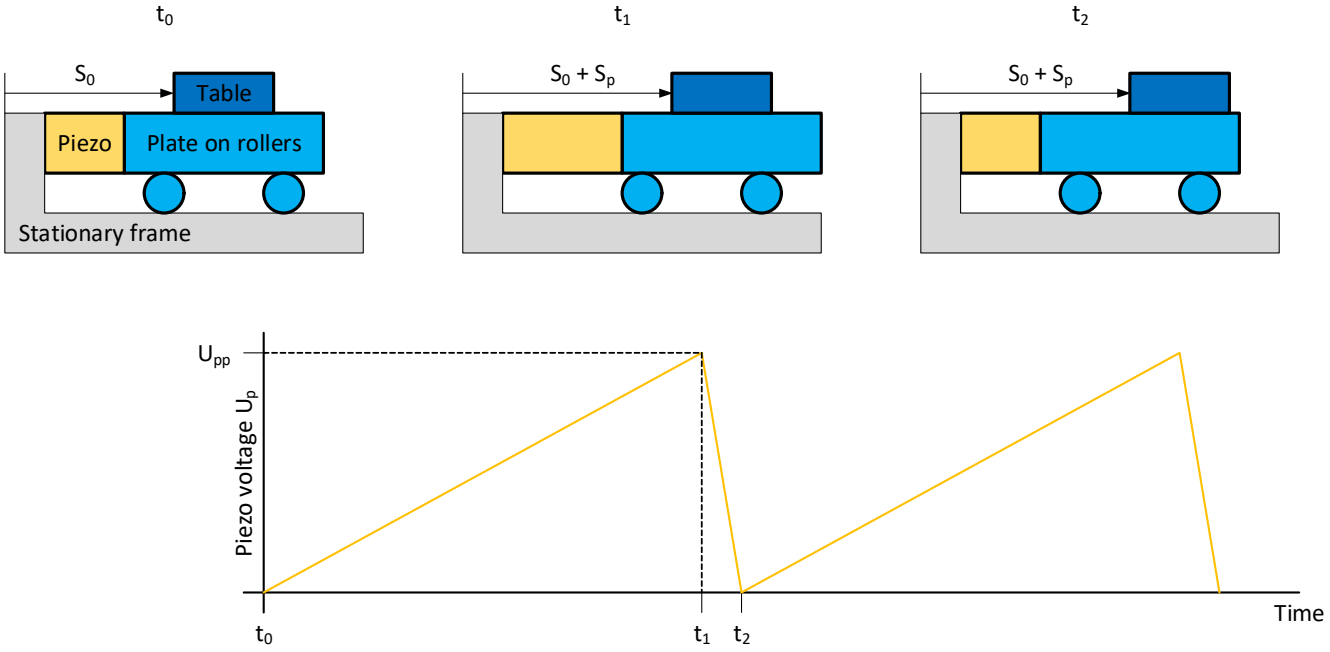



*Fig 1: Stick-slip positioners and sawtooth drive signal*

## 2. Power loss mechanisms (sawtooth signal)

### 2.1 Energy loss per step

The lost energy per step of a positioner is the sum of the losses in the piezo, friction and cabling. Details can be found in 2.1.1 to 2.1.3.

$$E_{step} = E_{piezo} + E_{friction} + E_{cable}$$

$$E_{step} = tan\delta C{U_{pp}}^2 + F_h S_p + C^2{U_{pp}}^2 R\left[\frac{1}{t_f} + \frac{1}{t_s}\right]$$

### 2.1.1 Piezo loss per step

A piezo behaves as a capacitance C and when charged it contains energy E. A fraction $E_{piezo}$ of this energy is lost in the piezo during both the charge and discharge cycle (1 full step). A measure of this loss is the material loss factor tanδ.

$$E = \frac{1}{2}C{U_{pp}}^2$$

$$E_{piezo} = 2\frac{1}{2}tan\delta C{U_{pp}}^2 = tan\delta C{U_{pp}}^2 \quad (tan\delta = loss\ factor)$$

### 2.1.2 Friction loss per step

Stick-slip positioners will generate heat in the friction contact between plate and table:

$$F_h = holding\ force$$

$$S_p = piezo\ stroke$$

$$E_{friction} = F_h S_p$$

### 2.1.3 Cable loss per step

Drive currents will cause resistive energy losses in wiring to the piezo with resistance R. For a sawtooth signal:

- Fast ramp, duration $t_f$

$$I_f = C\frac{U_{pp}}{t_f}$$

$$P_f = {I_f}^2 R$$

$$E_{cable_f} = {I_f}^2 R t_f = C^2 \frac{{U_{pp}}^2}{t_f} R$$

- Slow ramp, duration $t_s$

$$I_s = C\frac{U_{pp}}{t_s}$$

$$P_s = {I_s}^2 R$$

$$E_{cable_s} = {I_s}^2 R t_s = C^2 \frac{{U_{pp}}^2}{t_s} R$$

- Total cable loss per step

$$E_{cable} = E_{cable_f} + E_{cable_s} = C^2 {U_{pp}}^2 R\left[\frac{1}{t_f} + \frac{1}{t_s}\right]$$

## 2.2 Energy loss per covered distance

In chapter 2.1 the dissipated energy $E_{step}$, per step $S_p$ of a stick-slip positioner driven with a sawtooth signal was calculated to be

$$E_{step} = tan\delta C {U_{pp}}^2 + F_h S_p + C^2 {U_{pp}}^2 R\left[\frac{1}{t_f} + \frac{1}{t_s}\right]$$

When one needs to cover a distance $S_{distance}$ a series of steps is needed and the dissipated energy becomes

$$E_{distance} = \frac{S_{distance}}{S_p} E_{step}$$

$$E_{distance} = \frac{S_{distance}}{S_p}\left[tan\delta C {U_{pp}}^2 + F_h S_p + C^2 {U_{pp}}^2 R\left[\frac{1}{t_f} + \frac{1}{t_s}\right]\right]$$

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## 2.3 Energy loss at given speed

In chapter 2.1 the dissipated energy $E_{step}$, per step $S_p$ of a stick-slip positioner driven with a sawtooth signal was calculated to be

$$E_{step} = tan\delta C{U_{pp}}^2 + F_h S_p + C^2{U_{pp}}^2 R\left[\frac{1}{t_f}+\frac{1}{t_s}\right]$$

When one needs to move at speed v a number of steps per second f is needed and the dissipated power becomes:

$$f = \frac{v}{S_p}$$

$$P_{speed} = fE_{step}$$

$$P_{speed} = f\left[tan\delta C{U_{pp}}^2 + F_h S_p + C^2{U_{pp}}^2 R\left[\frac{1}{t_f}+\frac{1}{t_s}\right]\right]$$

# 3. Piezo operation at cryogenic temperatures

Material parameters have been determined for each type of piezo used in the analysis of chapter 4, based on datasheets and additional data on the internet. The specific values will be mentioned in the analysis, but the following basic relations largely apply:

**Loss factor**
Depending on the drive voltage different loss factors apply. On datasheets often small signal values are given, which are only valid up to an estimated 15 % of the maximum allowed drive voltage Upp. Large signal loss factor values are typically a factor 7 larger and used when driving at 100% of Upp. For operation at intermediate signals, 25% of Upp, an average of the small and large signal loss factor is assumed. Loss factor values for shear piezos are approximately a factor 2 larger than longitudinal stack piezos. At cryogenic temperatures values drop by a factor ≈2.5.

**Stroke**
The piezo stroke is linear with driving voltage and reaches a maximum at Upp. However, at cryogenic temperatures the stroke to voltage sensitivity will decrease significantly and the maximum piezo stroke decreases accordingly. Piezo stroke reduction values up to a factor 10 are mentioned at cryogenic temperatures. However, in practice a reduction of factor 4 is more typical, which might be explained by self-heating of the piezo ceramic at dynamic operation.

**Capacitance**
Like the loss factor, small signal values are usually given on datasheets. Large signal values are typically a factor 1.7 larger than small signal values, and intermediate signal values are assumed to be the average of the small and large signal values. At cryogenic temperatures values drop by a factor ≈4.5.

# 4. Benchmark comparison of 3 piezo types

For the dissipation analysis only a limited set of benchmark values at 4 K is needed:

- Sawtooth drive signal:
  - Fast ramp with duration $t_f$ = 10 µs.
  - The slow ramp duration $t_s$ = (1/f- $t_f$), with f the driving frequency (steps per second).
- Positioner speed: 0.1 mm/s.
- The impact of the applied piezo voltage is analyzed by calculating dissipation for 3 voltage levels:
  - Large Signal (LS) operation: drive voltage is 100% Upp, the step frequency is set to match the specified speed. Large signal values are used for the loss factor and capacitance.
  - Intermediate Signal (IS) operation: drive voltage is 25% of Upp, intermediate signal values are used for the loss factor and capacitance. The frequency is increased to maintain the specified speed.
  - Small Signals (SS) operation: drive voltage is 15% of Upp, small signal values are used for the loss factor and capacitance. The frequency is increased further to maintain the specified speed.

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- Loss of motion is 50 nm. Finite structural stiffnesses will cause part of the piezo motion to be lost in elastic micro deformations throughout the positioner and in the piezo itself. It is as if the piezo stroke is reduced by this value.
- Holding force between plate and table $F_h$= 3 N.
- Cable resistance to the piezo R = 1 Ω.
    - This is equivalent to approximately 0.3 m of low thermal conductivity phosphor bronze wire, or several hundred meters of copper wire.

| | | | Piezo 1: PICMA | Piezo 2: Plate stack | Piezo 3: PICA |
|---|---|---|---|---|---|
| **Piezo characteristics** | | | | | |
| **Supplier** | | | PI Ceramic | Noliac | PI Ceramic |
| **Product code** | | | P-883.11 | NAC2002-H08 | P-111.03T |
| **Material** | | | PIC252 | NCE51 | PIC255 |
| **Type** | | | Longitudinal stack | Longitudinal stack | Shear stack |
| **Footprint** | A x B | | 3 x 3 mm | 3 x 3 mm | 3 x 3 mm |
| **Height** | H | | 9 mm | 8 mm | 4,4 mm |
| **Max pkpk voltage** | $U_{pp}$ | | 150 V (-30 V to +120 V) | 60 V | 500 V (±250) |
| **Stroke at $U_{pp}$, 300 K** | $S_{p300}$ | | 10 µm | 8.6 µm | 3 µm |
| **Stroke at $U_{pp}$, 4 K** | $S_{p4}$ | | 2.50 µm | 2.15 µm | 0.75 µm |
| **SS loss factor 300 K** | $\tan\delta_{s300}$ | | 0.020 | 0.015 | 0.04 |
| **LS loss factor 300 K** | $\tan\delta_{l300}$ | | 0.14 | 0.105 | 0.28 |
| **SS loss factor 4 K** | $\tan\delta_{s4}$ | | 0.008 | 0.006 | 0.016 |
| **IS loss factor 4 K** | $\tan\delta_{i4}$ | | 0.032 | 0.024 | 0.064 |
| **LS loss factor 4 K** | $\tan\delta_{l4}$ | | 0.056 | 0.042 | 0.112 |
| **SS capacitance 300 K** | $C_{s300}$ | | 210 nF | 1080 nF | 1.5 nF |
| **LS capacitance 300 K** | $C_{l300}$ | | 357 nF | 1836 nF | 2.55 nF |
| **SS capacitance 4 K** | $C_{s4}$ | | 47 nF | 240 nF | 0.33 nF |
| **IS capacitance 4 K** | $C_{i4}$ | | 63 nF | 324 nF | 0.45 nF |
| **LS capacitance 4 K** | $C_{l4}$ | | 79 nF | 408 nF | 0.57 nF |
| **Positioner characteristics** | | | | | |
| **Loss of motion** | $S_{vp}$ | | 0.05 µm | 0.05 µm | 0.05 µm |
| **Effective positioner step, 4 K** | $S_{p4_eff}$ | LS<br>IS<br>SS | 2.45 µm<br>0.575 µm<br>0.325 µm | 2.10 µm<br>0.488 µm<br>0.273 µm | 0.70 µm<br>0.138 µm<br>0.063 µm |
| **Positioner holding force** | $F_h$ | | 3N | 3N | 3N |
| **Drive frequency at 0.1 mm/s, 4 K** | f | LS<br>IS<br>SS | 41 Hz<br>174 Hz<br>308 Hz | 48 Hz<br>205 Hz<br>367 Hz | 143 Hz<br>727 Hz<br>1600 Hz |
| **Fast ramp time** | $t_f$ | | 10 µs | 10 µs | 10 µs |
| **Slow ramp time** | $t_s$ | | 1/f- $t_f$ | 1/f- $t_f$ | 1/f- $t_f$ |
| **Cable resistance** | $R_c$ | | 1 Ω | 1 Ω | 1 Ω |
| **Dissipation per step at 4K** | | | | | |
| **Piezo loss per step** | $E_{piezo}$ | LS<br>IS<br>SS | 88.9 µJ<br>2.6 µJ<br>0.2 µJ | 88.1 µJ<br>2.4 µJ<br>0.1 µJ | 16.0 µJ<br>0.5 µJ<br>0.03 µJ |
| **Friction loss per step** | $E_{friction}$ | LS<br>IS<br>SS | 7.4 µJ<br>1.7 µJ<br>1.0 µJ | 6.3 µJ<br>1.5 µJ<br>0.8 µJ | 2.1 µJ<br>0.4 µJ<br>0.2 µJ |
| **Cable loss per step (fast ramp + slow ramp)** | $E_{cable}$ | LS<br>IS<br>SS | 14.0 + 0.0 = 14.0 µJ<br>0.6 + 0.0 = 0.6 µJ<br>0.1 + 0.0 = 0.1 µJ | 60.0 + 0.0 = 60.0 µJ<br>2.4 + 0.0 = 2.4 µJ<br>0.5 + 0.0 = 0.5 µJ | 8.1 + 0.0 = 8.1 µJ<br>0.0 + 0.0 = 0.0 µJ<br>0.0 + 0.0 = 0.0 µJ |
| **Total loss per step** | $E_{step}$ | LS<br>IS<br>SS | 110,3 µJ<br>4.9 µJ<br>1.3 µJ | 154,4 µJ<br>6.2 µJ<br>1.4 µJ | 18.1 µJ<br>0.9 µJ<br>0.2 µJ |
| **Positioner dissipation at 4K** | | | | | |
| **Total loss per covered distance (piezo + friction + cable)** | $D_{estituted}$ | LS<br>IS<br>SS | 36.28 + 3.00 + 5.73 = 45.01 mJ/mm<br>4.47 + 3.00 + 0.97 = 8.44 mJ/mm<br>0.59 + 3.00 + 0.35 = 3.93 mJ/mm | 41.97 + 3.00 + 28.55 = 73.52 mJ/mm<br>4.93 + 3.00 + 4.86 = 12.79 mJ/mm<br>0.43 + 3.00 + 1.72 = 5.15 mJ/mm | 22.80 + 3.00 + 0.012 = 25.81 mJ/mm<br>3.27 + 3.00 + 0.002 = 6.28 mJ/mm<br>0.48 + 3.00 + 0.001 = 3.48 mJ/mm |
| **Total loss at 0.1 mm/s speed (piezo + friction + cable)** | $P_{speed_total}$ | LS<br>IS<br>SS | 3.628 + 0.300 + 0.573 = 4.501 mW<br>0.447 +0.300 + 0.097 = 0.844 mW<br>0.059 + 0.300 + 0.035 = 0.393 mW | 4.197 + 0.300 + 2.855 = 7.352 mW<br>0.493 + 0.300 + 0.486 = 1.279 mW<br>0.043 + 0.300 + 0.172 = 0.515 mW | 2.280 + 0.300 + 0.001 = 2.581 mW<br>0.327 + 0.300 + 0.000 = 0.628 mW<br>0.048 + 0.300 + 0.000 = 0.348 mW |
| **Total loss at 0.25 mm/s speed (piezo + friction + cable)** | | LS<br>IS<br>SS | 9.069 + 0.750 + 1.433 = 11.252 mW<br>1.117 + 0.750 + 0.243 = 2.110 mW<br>0.146 + 0.750 + 0.086 = 0.983 mW | 10.491 + 0.750 + 7.138 = 18.379 mW<br>1.234 + 0.750 + 1.214 = 3.197 mW<br>0.107 + 0.750 + 0.430 = 1.287 mW | 5.700 + 0.750 + 0.003 = 6.453 mW<br>0.818 + 0.750 + 0.001 = 1.569 mW<br>0.119 + 0.750 + 0.001 = 0.869 mW |

*LS, IS, SS: see chapter 4*

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## 5. Conclusions and observations

- Increasing the step frequency while reducing the drive voltage is very effective to reduce the total dissipation per travelled distance or at given speed. The practical lower voltage limit is reached when a positioner starts to stall, or the targeted speed cannot be achieved with feasible drive frequencies.
    - Dissipated energy per mm E (mJ/mm) is independent of the speed, only the required number of steps per mm is relevant. A value of 1-2 mJ/mm is sometimes mentioned but this application note shows that such numbers are at the lower extreme of what seems feasible in practice and are probably not considering friction and cable losses. A more realistic value would be 4-5 mJ/mm.
    - Power dissipation P (mW) is proportional to speed: the number of steps per second changes but the dissipation per step remains the same. Power dissipation can be matched with available cooling power C (mW) of the cryostat. The maximum speed v (mm/s) to cover a distance x can be calculated by v = C/E.
- Higher voltage piezos like the PICMA and PICA have an advantage over the lower voltage Noliac when comparing their heat load to the environment, but differences are relatively small, especially when considering the limited accuracy of piezo characteristics in cryogenic conditions.
- Cable losses only occur during the fast ramp.
- Piezo dissipation is highly dominant at Large Signal operation, but the other contributions become significant at Intermediate Signal operation. At Small Signal operation friction becomes dominant.
    - Avoiding this friction is the goal of an alternative to the stick-slip concept. However, these so-called walkers require a multitude of piezos, up to 8, and their contribution will (partially) counteract the expected reduction of dissipation. The well-known inchworm is an example of such a walker.

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