# PRECISION POINT

Engineering Fundamentals

## BUCKLING FUNDAMENTALS

### Introduction

For slender structures there is a risk that buckling may occur when loaded with compression. This sheet gives an overview of how to interpret buckling and how to identify the potential risks. See <u>Beam theory: Buckling</u> to calculate buckling loads.

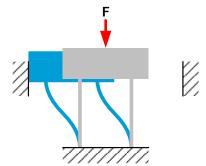
### What is buckling?

Buckling is often seen as a failure mode of the material in which at a certain threshold force suddenly large deformations are induced (e.g., when a soda can is compressed and collapses). A more accurate definition is that buckling is an instability in a structure, which is the result of the buckling shape having a lower potential energy. In the case of a beam loaded in compression with a force equal to the buckling force (see figure below), the structure can switch easily from the grey shape to the blue shape because the potential energy of the blue shape is lower. This does not mean that the blue shape will fail (see section failure modes).

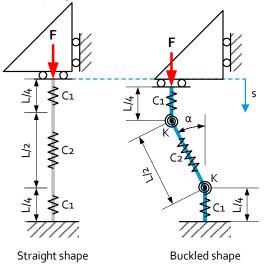


#### Instability in detail

As an example, to show the instability in a system, we consider the flexures from a parallel guiding mechanism. Due to the different boundary conditions to the previous example, the buckling shape will look like an S-curve.



This curve is approached by simplifying the flexure to three straight beams (which have axial stiffness) connected by two torsion springs:



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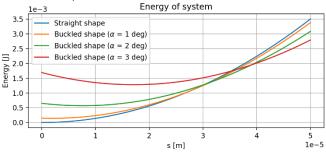
We can calculate the potential energy of both shapes using the following equations:

 $\begin{array}{l} P_{straight} = P_{compression} \\ P_{buckled} = P_{compression} + P_{rotation} \end{array}$ 

$$\begin{split} P_{compression} &= \frac{1}{2} C_{eq} s^2 \text{ where } C_{eq} \text{ indicates the equivalent} \\ \text{stiffness of } C_1 \text{ and } C_2 \text{ for either the straight or buckled shape.} \\ P_{rotation} &= \frac{1}{2} (K + K) \alpha^2 \end{split}$$

The potential energy as function of the displacement s for the straight shape and for 3 variants of the buckled shape (different values of  $\alpha$ ) is shown in the graph below. It can be seen that at s = 0 all 3 variants of the buckled shape have higher potential energy than the straight shape. This is mainly due to the energy stored in the torsion springs ( $P_{rotation} > 0$ ). When we look at higher values of s, the buckled shape curves will eventually cross the curve of the straight shape and will end up with lower potential energy.  $P_{rotation}$  will be constant for higher values of s while  $P_{compression}$  will increase, apparently faster for the straight shape than for the buckled shape. This is a result of the lower compression stiffness  $C_{eq}$  of the buckled shape.

To summarize, we can see that the buckled shapes initially have higher potential energy than the straight shape, but at higher displacement eventually have lower potential energy. This explains why buckling occurs at a certain threshold displacement or force.



#### Failure modes

We now know that buckling is an instability in the mechanism that can have large deformations as a result. These large deformations can result in material failure due to high stresses, but this is not always the case. Often the function of a mechanism can no longer be fulfilled when buckling occurs, which is a different failure mode.

For example, see again the parallel guiding with end stops discussed before: At a certain load the parallel guiding cannot support the load and the flexures will buckle, moving the top block against one of the end stops. The failure mode here is that the function of the mechanism, which is supporting a load in its nominal position, can no longer be fulfilled. However, if the end stop is close enough to the nominal position of the top block, it will stop the material from failing. Consequently, the mechanism returns to its original state when the load is removed.