Introduction

This sheet describes the basics of electrical network analysis. The description focuses on the modeling of a circuit for static or dynamic analysis using nodal analysis. Calculation may be performed with a computer aid such as MATLAB.

A network of passive elements (such as resistance and/or reactance) can be analyzed on current and voltage in each circuit node as result of used component parameters, voltage- and current sources.

Background

- A 'node' is a point ('star') between components
- Symbols: voltage (u), current (i), resistance (R)
- Using Ohm's law -> $G \cdot u = i$ with $G = \frac{1}{R}$
- Sum of currents in each node of circuit is zero: $\sum i = 0$

Modeling Procedure

1) Set up a network without sources; sources are replaced by open circuits. All node voltages are referenced to ground voltage. As the potential of the reference node is known, this node is left out of the equation.
2) The circuit model is described in a set of equations in a partitioned matrix:
   - according to Ohm's law: $i_s = G \cdot \bar{u} + Q$
   - node voltage constraints: $u_s = P \cdot \bar{u}$

   resulting in: $\begin{bmatrix} \boldsymbol{G} & \boldsymbol{Q} \\ \boldsymbol{P} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{\imath} \end{bmatrix} = \begin{bmatrix} i_s \\ u_s \end{bmatrix}$
3) Matrix G is the model description without sources:
   - On diagonal $\boldsymbol{G}_{i,i}$; passives connected to node u_i. (positive values)
   - Other matrix elements $\boldsymbol{G}_{i,j}$: passives connected to other nodes from u_i to u_j (negative values)

   Crosscheck:
   Sum of row $\boldsymbol{G}_i$ results in elements connected to reference from node u_i
   Sum of col $\boldsymbol{G}_j$ results in elements connected to reference from node u_j
4) Matrix Q describes the current constrains for voltage sources. Matrix P describes the voltage constrains between nodes (or relationship).
5) Current sources may be added in vector $\boldsymbol{i_s}$ and voltage sources in vector $\boldsymbol{u_s}$

Resulting unknowns

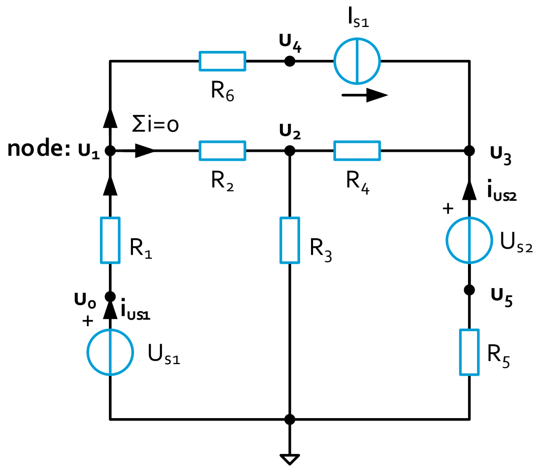
From this set of equations differential voltages and currents can be calculated:

- Differential voltages: $u_{xy} = u_x - u_y$
- Currents: $i_{xy} = \dfrac{u_{xy}}{R_{xy}}$

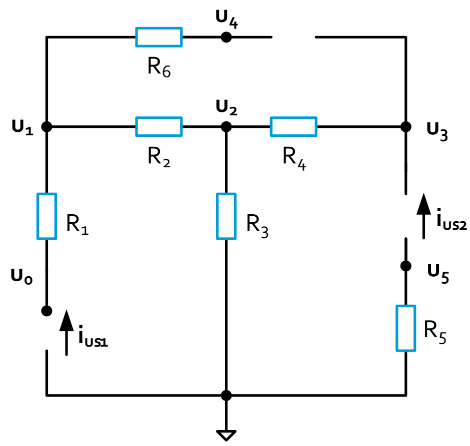
Dynamic analysis

Each passive may be described as resistance R, inductive reactance $X_L = j\omega L$ or capacitive reactance $X_c = \frac{1}{j\omega C}$. Keep in mind that the components in the matrix are inverted so: Conductance: $G = \frac{1}{R}$ or susceptance: $B = \frac{1}{X}$. For dynamic analysis only the source of interest has to be described, other sources are short or open for voltage and current sources respectively.

Example: Circuit with voltage- and current sources



Modeling matrix $\boldsymbol{G}$ by neglecting sources:



For passives; $G = \frac{1}{R}$ thus for example: $G_1 = \frac{1}{R_1}$

$$\left[\begin{array}{cccccc:cc} G1 & -G1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -G1 & (G1+G2+G6) & -G2 & 0 & -G6 & 0 & 0 & 0 \\ 0 & -G2 & (G2+G3+G4) & -G4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G4 & G4 & 0 & 0 & 0 & 1 \\ 0 & -G6 & 0 & 0 & G6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G5 & 0 & -1 \\ \hdashline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \end{array}\right] \left[\begin{array}{c} u0 \\ u1 \\ u2 \\ u3 \\ u4 \\ u5 \\ \hdashline i_{us1} \\ i_{us2} \end{array}\right] = \left[\begin{array}{c} i_s \\ \hdashline u_s \end{array}\right]$$

After establishing the description of the network in matrix notation, the sources can be added.

Now we expand the equation by adding the voltage source descriptions $u_{s2} = u_3 - u_5$, $u_{s1} = u_0$ and current source: node u_3 current is sourced with $+i_{s1}$ and in node u_4 current is sourced with $-i_{s1}$ (thus sinked).

$$\left[\begin{array}{cccccc:cc} G1 & -G1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -G1 & (G1+G2+G6) & -G2 & 0 & -G6 & 0 & 0 & 0 \\ 0 & -G2 & (G2+G3+G4) & -G4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G4 & G4 & 0 & 0 & 0 & 1 \\ 0 & -G6 & 0 & 0 & G6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G5 & 0 & -1 \\ \hdashline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \end{array}\right] \left[\begin{array}{c} u0 \\ u1 \\ u2 \\ u3 \\ u4 \\ u5 \\ \hdashline i_{us1} \\ i_{us2} \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ i_{s1} \\ -i_{s1} \\ 0 \\ \hdashline u_{s1} \\ u_{s2} \end{array}\right]$$

Vector $[\bar{u} \quad \bar{\imath}]^T$ may be calculated by:

- using: $\begin{bmatrix} \bar{u} \\ \bar{\imath} \end{bmatrix} = \begin{bmatrix} G & Q \\ P & 0 \end{bmatrix}^{-1} \begin{bmatrix} i_s \\ u_s \end{bmatrix}$
- or using the reduced row echelon form of matrix $\begin{bmatrix} G & Q & i_s \\ P & 0 & u_s \end{bmatrix}$