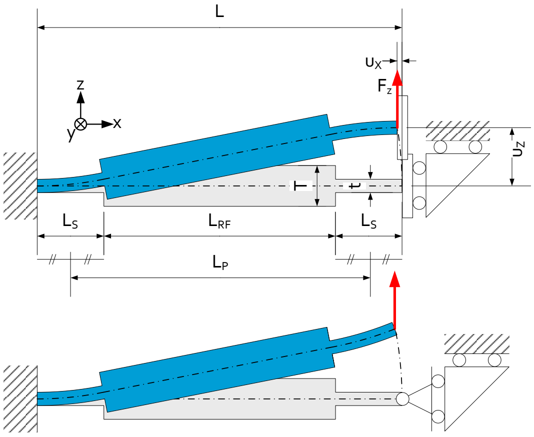
Introduction

Flexures or leaf springs can be used for play and friction free motion. A downside is stiffness and to minimize the needed force, the flexures are made slender and thin. Smart reinforcement of flexures and leaf springs can help to keep the needed motion-force minimal while the flexure or leaf spring is made thicker, which is beneficial for its carrying stiffness and easier to manufacture which will decrease the manufacturing costs.



S-shape deformation (top) and c-shape deformation (bottom) of a reinforced leaf spring with y-dimension b.

Design parameters

$\lambda = \frac{L_S}{L}$ $\qquad$ $\gamma = \frac{t}{T}$

$0 < \lambda < \frac{1}{2}$ $\qquad$ $0 < \gamma < 1$

$L_P = L_{RF} + L_s = (1 - \lambda)L$

Deformation characteristics

$$u_z = \frac{F_z}{C_z}$$

$$u_x = \frac{u_z^2}{2L(1-\lambda)} \qquad \text{(s-shape deformation)}$$

Stiffness for s- and c-shape deformation

$$C_x = \frac{1}{2\lambda(1-\gamma)+\gamma} \cdot \frac{Etb}{L}$$

$$C_y = \frac{1}{2\lambda(4\lambda^2-6\lambda+3)(1-\gamma)+\gamma} \cdot \frac{Etb^3}{L^3} \qquad \text{(s-shape deformation)}$$

$$C_y = \frac{1}{2\lambda(4\lambda^2-6\lambda+3)(1-\gamma)+\gamma} \cdot \frac{Etb^3}{4L^3} \qquad \text{(c-shape deformation)}$$

$$C_z = \frac{1}{2\lambda(4\lambda^2-6\lambda+3)(1-\gamma^3)+\gamma^3} \cdot \frac{Ebt^3}{L^3} \qquad \text{(s-shape deformation)}$$

$$C_z = \frac{1}{2\lambda(4\lambda^2-6\lambda+3)(1-\gamma^3)+\gamma^3} \cdot \frac{Ebt^3}{4L^3} \qquad \text{(c-shape deformation)}$$

$$K_x = \frac{1}{2\lambda(1-\gamma^3)+\gamma^3} \cdot \frac{Gbt^3}{3L}$$

$$K_y = \frac{1}{2\lambda(1-\gamma^3)+\gamma^3} \cdot \frac{Ebt^3}{12L} \qquad \text{(c-shape deformation)}$$

$$K_z = \frac{1}{2\lambda(1-\gamma)+\gamma} \cdot \frac{Etb^3}{12L} \qquad \text{(c-shape deformation)}$$

Force limits (buckling)

When a force in x-direction is applied buckling can occur, for equations to calculate the buckling load see *Beam Theory: Buckling*.

Design guidelines

Keep $\quad \frac{1}{10} < \lambda < \frac{1}{3} \quad$ and $\quad \frac{1}{10} < \gamma < \frac{1}{2}$

Typical $\quad \lambda = \frac{1}{6} \quad$ and $\quad \gamma = \frac{1}{5}$

Then:

$$C_x = 2.1 \cdot \frac{Etb}{L} \qquad K_x = 3.0 \cdot \frac{Gbt^3}{3L}$$

$$C_y = 1.3 \cdot \frac{Etb^3}{L^3} \text{ (s-shape/c-shape)} \qquad K_y = 3.0 \cdot \frac{Ebt^3}{12L} \text{ (c-shape)}$$

$$C_z = 1.4 \cdot \frac{Ebt^3}{L^3} \text{ (s-shape/c-shape)} \qquad K_z = 2.1 \cdot \frac{Etb^3}{12L} \text{ (c-shape)}$$

Normalized stiffness increase due to reinforcement

The graphs shown below indicate the normalized stiffness increase with respect to the non-reinforced case ($\lambda = 0.5$).