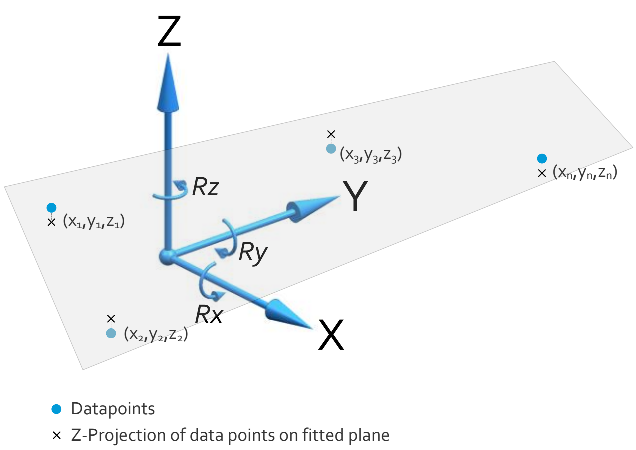
FIT PLANE THROUGH POINTS

Introduction

Method to derive a best fit a plane through number (≥ 3) XYZ data points, where the summed square errors of the data points in relation to the fit-plane in Z-direction is minimal.



Equation of a plane

$$z(x,y) = A \cdot x + B \cdot y + C$$

Data points

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \\ & \vdots & \\ x_n & y_n & z_n \end{bmatrix}$$

Coefficients of plane equation

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum_{i=1}^{n} x_i z_i \\ \sum_{i=1}^{n} y_i z_i \\ \sum_{i=1}^{n} z_i \end{bmatrix}$$

Tip / Tilt angles

$$\mathrm{Rx} = \mathrm{atan}\left[\frac{d}{dy} z(x,y)\right] = \mathrm{atan}[B]$$

$$\mathrm{Ry} = \mathrm{atan}\left[-\frac{d}{dx} z(x,y)\right] = \mathrm{atan}[-A]$$

Fit quality – Coefficient of determination = R^2

$$R^2 = 1 - \frac{\sum_{i=1}^{i=n}\left(z_i - z(x_i, y_i)\right)^2}{\sum_{i=1}^{i=n}\left(z_i - \frac{1}{n}\sum_{i=1}^{n} z_i\right)^2}$$

A value of R^2 which is close to 1 indicates a good fit quality.